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NEW YORK UNIVERSITY
Courant Institute of Mathematical Sciences
Division of Electromagnetic Research

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FINAL REPORT

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FINAL REPORT

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Project 8600
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November 30, 1959 - December 1, 1962

MAGNETOHYDRODYNAMICS

Prepared
for

HQ Air Force Cambridge Research Laboratories
Air Force Research Division
United States Air Force
Cambridge, Massachusetts

December, 1962

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Summary

During the period beginning November 30, 1959 and ending December 1, 1962 the magneto-hydrodynamics group of the Division of Electromagnetic Research, Courant Institute of Mathematical Sciences at New York University has carried on research in hydromagnetic theory under Contract No. AF 19 (604) 6144. In accordance with the contract, the objective has been to conduct studies and mathematical investigations of the motion of charged particles of an ionized fluid in the presence of external electric and magnetic fields, with particular applications to motions in the ionized layers of the atmosphere and to the influence such motions have on the geomagnetic field at or near the earth's surface.

In Section 1, the work of Drs. J. Bazer, W.B. Ericson and J. Hurley is summarized. Abstracts of completed work are given in Section 2. Section 3 comprises a list of reports and publications; conferences attended are listed in Section 4.

The work of the following faculty members and research associates was supported by contract funds: Prof. J. Bazer, Dr. H. Hochstadt, Dr. J. Hurley, and Dr. R. Lewis.

1. Non-Linear and Linearized Hydromagnetic Flow - J. Bazer, W.B. Ericson, J. Hurley

It was observed by K.O. Friedrichs several years ago that the hydromagnetic equations for a perfect, infinitely conducting, compressible fluid share with the equations of gas dynamics the property of being a first order "symmetric hyperbolic" system of "conservation laws." This observation meant that the equations of magnetohydrodynamics could be treated by methods similar to those employed in ordinary gas dynamics. Specifically, it meant, at least in principle, that i) one could develop a theory of one-dimensional* hydromagnetic simple waves which together with hydromagnetic shocks could be used to solve initial value problems with sufficiently simple conditions; ii) one could develop a theory of geometrical hydromagnetics dealing with the propagation of small disturbances, similar to the existing theories of geometrical acoustics and optics; and finally iii) one could develop a theory of two-dimensional** steady hydromagnetic flow by employing suitable generalizations of methods used in the steady flow theory of gas dynamics. These three observations constituted the basis of a program initiated by us from the outset.

Our plans for work on one-dimensional simple waves and shocks have been successfully completed. In addition to the work described in detail in the Final Report of December, 1959 (Contract No. AF 19(604)2138) Drs. J. Bazer and W.B. Ericson have published a paper "On Certain Properties of Hydromagnetic Shocks" (Phys. of Fluids 3 (1960), 631) in which they extend the results of their report MH-8 on one-dimensional shocks. Drs. Bazer and Ericson have also extended the results of this report in another direction. Specifically, in MH-12, they apply them to the study of oblique shocks in steady two-dimensional hydromagnetic flow and give an analytical method for determining the strength and orientation of these shocks, given the flow speed in front and the turning angle of the velocity field. Other important characteristics of the flow are also described. The main emphasis in this work is on the aligned field case (velocity field parallel to the magnetic field); however, the general case is also briefly discussed. In MH-11, Prof. Bazer has extended the earlier work by O.W. Fleischman and himself (MH-10), dealing with the propagation of weak hydromagnetic discontinuities, to cover the reflection and refraction of such discontinuities at boundaries and at discontinuities in the underlying flow. A thoroughgoing analysis is made of the hydromagnetic analog of Snell's laws for all angles of incidence and for all orientations of the magnetic field. The coefficients of reflection and refraction of slow, fast and Alfvén waves at a hydromagnetic contact discontinuity and the reflection coefficients at a perfectly conducting rigid wall are obtained.

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* By one-dimensional hydromagnetic flow we mean a hydromagnetic flow in which the dependent variables, the magnetic intensity, fluid velocity, density, etc., are functions of one space variable and the time t; no restrictions are placed on the orientation of the magnetic and flow vectors.

** Let $\vec{B}$, $\vec{u}$, ρ be the magnetic induction, particle velocity and mass density, respectively. In two-dimensional steady flow $\vec{B}$, $\vec{u}$ and ρ depend on x and y alone and $\vec{B}$ and $\vec{u}$ are confined to the (x,y)-plane.

In another area of plasma physics Dr. J. Hurley has considered the problem of the transition region separating a plasma from its confining magnetic field. In a previous paper (Analysis of the Transition Region between a Plasma and its Confining Magnetic Field, Phys. Rev., 124 (1961), 1307),he had obtained an exact solution for the one problem, assuming that the electrons were at rest. In MH-13 he extends this analysis to include electron motion.

2. Completed Work (Abstracts)

2.1 On Certain Properties of Hydromagnetic Shocks - W.B. Ericson* and J. Bazer

Proofs of four basic properties of stationary, planar, non-relativistic hydromagnetic shocks are presented. These properties are:

(P_1) the specific entropy behind a hyrdomagnetic shock exceeds that ahead, if and only if, the shock is compressive;

(P_2) the specific entropy behind a compressive shock varies in the same sense as the mass flux;

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(P_3) in the region behind (compressive) fast shocks, the fast disturbance speed is greater than the normally-directed fluid velocity relative to the shock;

(P_4) in the region behind (compressive) slow shocks, the slow disturbance speed may be less than, equal to, or greater than the normally-directed fluid velocity relative to the shock. The equality holds when the specific entropy and the mass flux assume their maximum values.

2.2 MH-11 Reflection and Refraction of Weak Hydromagnetic Discontinuities - J. Bazer

Abstract

The problem of reflection and refraction of weak hydromagnetic disturbances at surfaces of discontinuity in the medium is studied. It is shown that the introduction of an appropriate set of direction angles for the unperturbed magnetic field on each side of the discontinuity makes possible (1) a simplification of V.C.A. Ferraro's laws relating to the reflection and refraction of Alfvén waves and (2) a simple graphical method for determining the real angles and speeds at which the reflected and refracted waves emerge. The relationship of the graphical procedure to Huyghens' construction is discussed. Application of the theory is made to several two-dimensional steady-flow problems and to the problems of reflection and refraction at a rigid infinite conductor and at a contact discontinuity.

2.2 MH-12 Oblique Shock Waves in Steady Two-Dimensional Hydromagnetic Flow - J. Bazer and W.B. Ericson*

Abstract

A method is presented for determining the orientation and strength of oblique-shock waves in steady two-dimensional (planar) hydromagnetic flow. The medium is assumed to be an infinitely conducting, nonviscous, ideal polytropic gas. The method employs the results of an earlier paper by the authors in which it was shown that the velocities relative to the shock, the excess density ratio, excess pressure ratio, etc., can be expressed as simple algebraic functions of h, the magnitude of the jump in magnetic field across the shock, and the angle, ω, which the shock makes with upstream field.

Attention is focused chiefly on the aligned fields case where the results are especially easy to assess and are directly

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applicable to problems involving flow around wedges and corners. In this case the ranges for the slow and fast shock angles are easily delimited in terms of the turning angle and the upstream sound to Alfvén speed ratio. Moreover, the dependence of the shock angle, ω on h, and hence of all the characteristics of the downstream flow, are readily determined in terms of the characteristics of the upstream flow.

2.3 MH-13 Analysis of the Transition Region between a Uniform Plasma and its Confining Magnetic Field: II - J. Hurley

Abstract

The problem considered is that of determining the structure of the transition region separating a uniform plasma from its confining magnetic field. The geometry chosen is one-dimensional. The magnetic field profile within the boundary layer is determined by the current distribution i.e., by the paths of the ions and electrons. The paths of these particles on the other hand are determined by the fields in which they move. An exact, self-consistent solution of Maxwell's equations and the equations of motion of the particles is obtained. A detailed analysis is made for a hydrogen plasma. These results are then compared with those of a plasma whose positive constituent is the proton and whose negative constituent has a mass which approaches zero and a speed which approaches infinity in such a way that the energy remains finite. The transition layer thickness for these two plasmas agree to one part in 10^6. It is shown that this latter case is equivalent to treating the electrons of the hydrogen plasma in the guiding center approximation.

3. Research Reports and Publications

Under Contract No. AF 19 (604)-926

MH-1	J.B. Keller	Electrohydrodynamics I. The Equilibrium of a Charged Gas in a Container Published - J. Rat. Mech. 5 (1956), 715	Jan. 1955
MH-2	B.A. Lippmann	The Decay of Magnetic Fields	May 1955
MH-3	B.A. Lippmann	The Decay of a Stellar Magnetic Field	June 1955
MH-4	O. Fleischman B.A. Lippmann	Remarks on Alfvén's Perturbation Method	Aug. 1955

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MH-5	J. Bazer	Resolution of an Initial Shear Flow Discontinuity in One-dimensional Hydromagnetic Flow Published - Astrophys. J. 128 (1958), 686	June 1955
MH-6	R.H. Kraichnan	Relation of Fourth to Second Moments in Stationary Homogeneous Hydromagnetic Turbulence Published - Phys. Rev. 107 (1957), 1485	Jan. 1957
MH-7	R.H. Kraichnan	Statistical Mechanics of Stationary Homogeneous Hydromagnetic Turbulence Published - Phys. Rev. 109 (1958), 1407	April 1957
MH-8	W.B. Ericson J. Bazer	Hydromagnetic Shocks Published - I. Astrophys. J. 129 (1959), 758 II. Phys. of Fluids 3 (1960), 631	Jan. 1958
MH-9	R.H. Kraichnan	The Structure of Isotropic Turbulence at Very High Reynolds Numbers Published - J. Fluid Mech. 5 (1959), 497	Feb. 1958
MH-10	J. Bazer O. Fleischman	Propagation of Weak Hydromagnetic Discontinuities Published - Phys. of Fluids 2 (1959), 366	May 1958
	J. Bazer W.B. Ericson	On Certain Properties of Hydromagnetic Shocks Published - Phys. of Fluids 3 (1960), 631 (Based in part on Research Report MH-8)	

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MH-11	J. Bazer	Reflection and Refraction of Weak Hydromagnetic Discontinuities	Feb. 1961
MH-12	J. Bazer W.B. Ericson	Oblique Shock Waves in Steady Two-dimensional Hydromagnetic Flow Published - Proc. Symp. on Electromagnetic and Fluid Dynamics of Gaseous Plasma, Polytechnic Inst. of Brooklyn, 11 (1961)	Dec. 1962
MH-13	J. Hurley	Analysis of the Transition Region between a Uniform Plasma and its Confining Magnetic Field: II Accepted - Phys. of Fluids (to appear, Feb. 1963)	Dec. 1962

4. Conferences Attended

Upon the invitation of Prof. D.C. Pack, Dr. Bazer delivered a series of lectures on Magnetogasdynamics at the Royal College of Science and Technology (Glasgow, Scotland), May 15 - June 15, 1961.

Drs. Bazer and Hurley attended the series of joint conferences in Washington, D.C., April - May, 1962, sponsored by the American Physical Society, the Americal Geophysical Union, and the Institute of Radio Engineers.

Dr. Hurley attended the recent conference of the Physical Society held at Atlantic City, November 28 - December 1, 1962.

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N. Y. U. Courant Institute of
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4 Washington Place
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